NAG Toolbox for MATLAB

s07aa

1 Purpose

s07aa returns the value of the circular tangent, $\tan x$, via the function name.

2 Syntax

```
[result, ifail] = s07aa(x)
```

3 Description

s07aa calculates an approximate value for the circular tangent of its argument, $\tan x$. It is based on the Chebyshev expansion

$$\tan\theta = \theta$$

$$y(t) = \theta \sum_{r=0}' c_r T_r(t)$$

where $-\frac{\pi}{4} < \theta < \frac{\pi}{4}$ and $-1 < t < +1$, $\quad t = 2\left(\frac{4\theta}{\pi}\right)^2 - 1$.

The reduction to the standard range is accomplished by taking

$$x = N\pi/2 + \theta$$

where N is an integer and $-\frac{\pi}{4} < \theta < \frac{\pi}{4}$,

i.e., $\theta = x - \left(\frac{2x}{\pi}\right)\frac{\pi}{2}$ where $N = \left[\frac{2x}{\pi}\right] =$ the nearest integer to $\frac{2x}{\pi}$.

From the properties of $\tan x$ it follows that

$$\tan x = \begin{Bmatrix} \tan\theta, & N\text{even} \\ -1/\tan\theta, & N\text{odd} \end{Bmatrix}$$

4 References

Abramowitz M and Stegun I A 1972 *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

5 Parameters

5.1 Compulsory Input Parameters

1: **x – double scalar**

The argument x of the function.

5.2 Optional Input Parameters

None.

5.3 Input Parameters Omitted from the MATLAB Interface

None.

5.4 Output Parameters

1: **result – double scalar**

The result of the function.

2: **ifail – int32 scalar**

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail $= 1$

The function has been called with an argument that is larger in magnitude than f_1; the default result returned is zero.

ifail $= 2$

The function has been called with an argument that is too close (as determined using the relative tolerance f_2) to an odd multiple of $\pi/2$, at which the function is infinite; the function returns a value with the correct sign but a more or less arbitrary but large magnitude (see Section 7).

7 Accuracy

If δ and ϵ are the relative errors in the argument and result respectively, then in principle

$$\epsilon \geq \frac{2x}{\sin 2x}\delta.$$

That is a relative error in the argument, x, is amplified by at least a factor $2x/\sin 2x$ in the result.

Similarly if E is the absolute error in the result this is given by

$$E \geq \frac{x}{\cos^2 x}\delta.$$

The equalities should hold if δ is greater than the ***machine precision*** (δ is a result of data errors etc.) but if δ is simply the round-off error in the machine it is possible that internal calculation rounding will lose an extra figure.

The graphs below show the behaviour of these amplification factors.

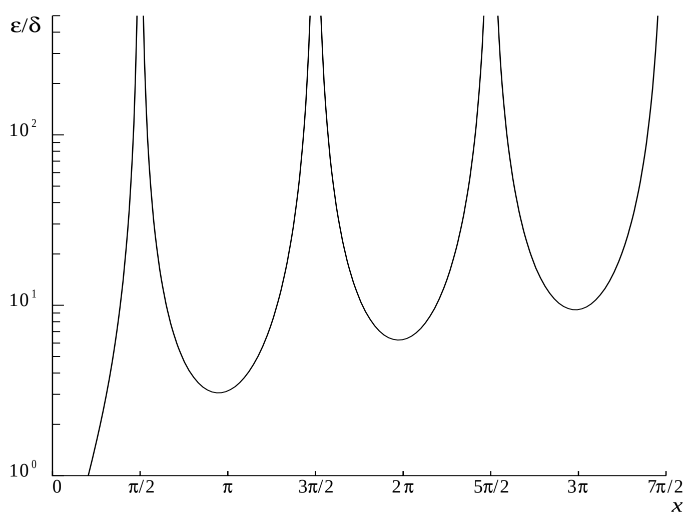

Figure 1

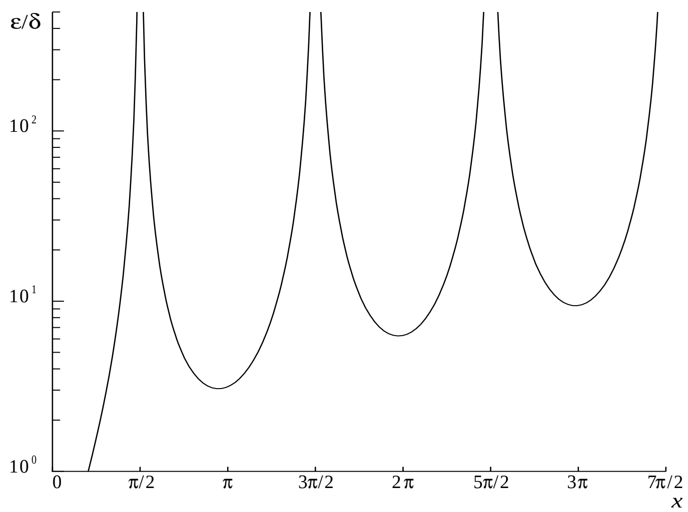

Figure 2

In the principal range it is possible to preserve relative accuracy even near the zero of $\tan x$ at $x = 0$ but at the other zeros only absolute accuracy is possible. Near the infinities of $\tan x$ both the relative and absolute errors become infinite and the function must fail (error 2).

If N is odd and $|\theta| \leq xF_2$ the function could not return better than two figures and in all probability would produce a result that was in error in its most significant figure. Therefore the function fails and it returns the value

$$-\operatorname{sign}\theta\left(\frac{1}{|xF_2|}\right) \simeq -\operatorname{sign}\theta\tan\left(\frac{\pi}{2} - |xF_2|\right)$$

which is the value of the tangent at the nearest argument for which a valid call could be made.

Accuracy is also unavoidably lost if the function is called with a large argument. If $|x| > F_1$ the function fails (error 1) and returns zero.

8 Further Comments

None.

9 Example

```
x = -2;
[result, ifail] = s07aa(x)
```

```
result =
    2.1850
ifail =
           0
```